For me context is the key - from that comes the understanding of everything. - Kenneth Noland

One-pass Context-based Tableaux Systems for CTL and ECTL.

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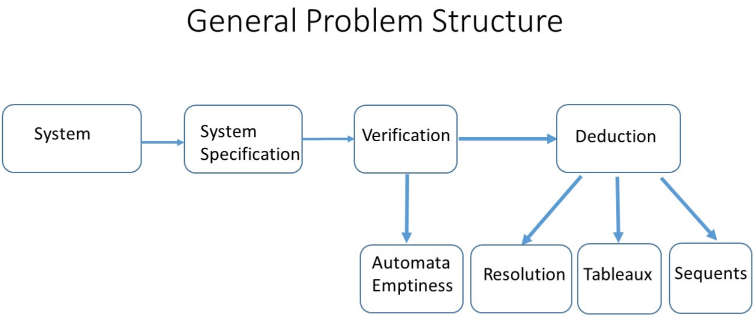
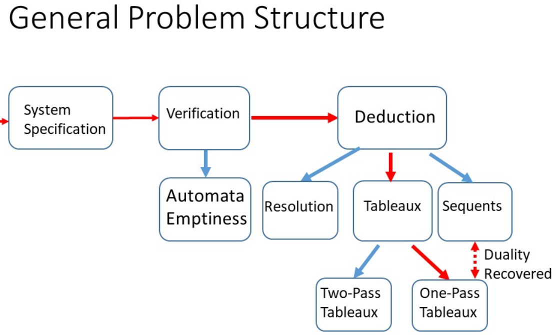
Structure of the Talk

- Motivation/General Problem Structure
- Branching-Time Logics Framework
- Tableaux: General Idea
- More motivation: 1 pass vs 2 pass Tableaux
- Our Approach - Context Based Tableaux
- Formalisms/Tableaux Rules for CTL
- Extension to ECTL
- Conclusions and Future Work

Motivation


General Problem Structure
System
System Specification
Verification
Deduction
Automata Emptiness
Resolution
Tableaux
Sequents

Motivation


General Problem Structure
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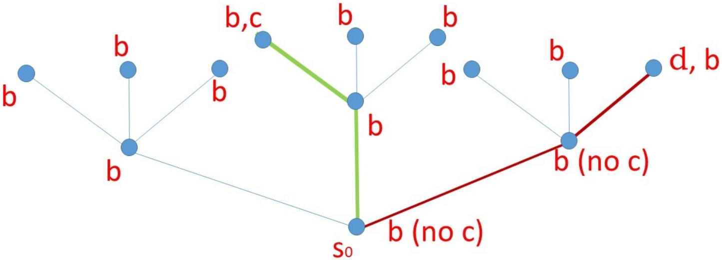
Tableaux
Sequents
Duality Recovered
Two-Pass Tableaux
One-Pass Tableaux

Branching-Time Framework

- Temporal operators:
  - $\bigcirc$ - ‘at the next moment of time’, $\diamondsuit$ - ‘eventually’, $\square$ - ‘always’ and $\mathcal{U}$ - ‘until’
- paths quantifiers: A - ‘for all paths’; E - ‘there exists a path

$S_0 \models A\square b$ $\qquad$ $S_0 \models E\diamondsuit c$ $\qquad$ $S_0 \models E(\neg c\,\mathcal{U} d)$



Tableaux: very general idea for one and two pass methods

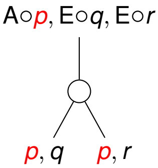
- Labelled graph
- Expansion rules: α rules generate a successor node $\dfrac{\phi \wedge \psi}{\phi, \psi}$
- β rules 'split' into alternative nodes $\dfrac{\phi \vee \psi}{\phi \mid \psi}$
- Use fixpoint properties of temporal operators:

  $\mathsf{A}\Box\phi = \phi \wedge \mathsf{A}\circ\mathsf{A}\Box\phi \qquad \mathsf{E}\phi\mathcal{U}\psi = \psi \vee (\phi \wedge \mathsf{E}\circ\mathsf{E}\phi\mathcal{U}\psi)$

$$\frac{\mathsf{A}\Box\phi}{\phi, \mathsf{A}\circ\mathsf{A}\Box\phi} \qquad \frac{\mathsf{E}\phi\mathcal{U}\psi}{\psi \mid \phi \wedge \mathsf{E}\circ\mathsf{E}\phi\mathcal{U}\psi}$$

Tableaux: building 'next' states

- A state is a node such that all its formulae are literals or of the form A○ϕ or E○ϕ
- This allows to build successor nodes to propagate ϕ.
- In our construction the tableaux graph is an AND-OR TREE:



Motivation: 2 pass vs 1 pass

- Two-pass tableaux:

1st pass: checks the validity of the given formula by creating a graph
2d pass: checks if all eventualities are satisfied.

- One-pass tableaux:

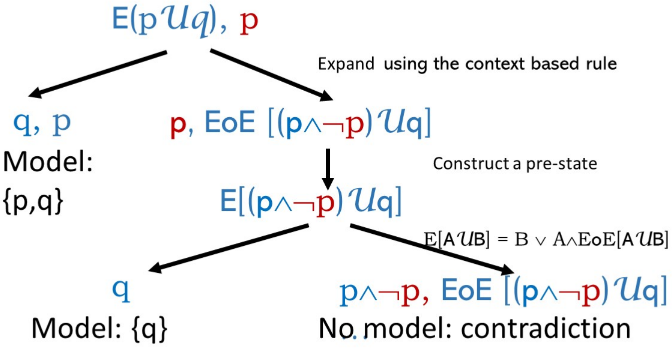
one pass: checks the validity of the input without auxiliary constructions,
need: mechanism to ensure this.

- Our mechanism to ensure a one pass - *CONTEXT* (of the selected eventuality)

Branching-Time Logics Framework: context

- CTL well formed formulae are state formulae - every temporal operator is preceded by a path quantifier:
  - $\mathsf{E}\Diamond p$ ✓
  - $\mathsf{E}\Box\Diamond p$ **X**
- Context
  - is a collection of state formulae accompanying the eventuality in the node.
  - forces eventualities to be fulfilled as soon as possible.
- In the node $\{\mathsf{E}(p\mathcal{U}q), p\}$ the context of $\mathsf{E}(p\mathcal{U}q)$ is p

CONTEXT forces fulfilment of eventualities



Branching-Time Logics Framework: CTL-type logics

State formulae

$$\sigma ::= \top \mid p \mid \sigma_1 \wedge \sigma_2 \mid \neg\sigma \mid \mathsf{E}\pi$$

Path formulae

$$\begin{array}{lll}
\pi_{\mathrm{CTL}^\star} & ::= & \sigma \mid \pi_1 \wedge \pi_2 \mid \neg\pi \mid \circ\pi \mid \pi\mathcal{U}\pi \\
\pi_{\mathrm{ECTL}^\#} & ::= & \sigma \mid \pi_1 \wedge \pi_2 \mid \neg\pi \mid \circ\sigma \mid \sigma\mathcal{U}(\sigma \wedge \Diamond\sigma) \\
 & & \quad \mid \Box(\sigma \vee \Box\sigma) \mid \sigma\mathcal{U}(\Box\sigma) \mid \Box(\sigma\mathcal{U}\sigma) \\
\pi_{\mathrm{ECTL}^+} & ::= & \sigma \mid \pi_1 \wedge \pi_2 \mid \neg\pi \mid \circ\sigma \mid \sigma\mathcal{U}\sigma \mid \Box\sigma \mid \Box\Diamond\sigma \mid \Diamond\Box\sigma. \\
\pi_{\mathrm{ECTL}} & ::= & \sigma \mid \neg\pi \mid \circ\sigma \mid \sigma\mathcal{U}\sigma \mid \Box\sigma \mid \Box\Diamond\sigma \mid \Diamond\Box\sigma. \\
\pi_{\mathrm{CTL}} & ::= & \sigma \mid \neg\pi \mid \circ\sigma \mid \sigma\mathcal{U}\sigma \mid \Box\sigma.
\end{array}$$

Development of One Pass Method for Branching Time

- Previous work - one pass tableaux for ECTL$^{\#}$
- This covers ECTL^{+}
- However, too complex to cover simpler logics - CTL and ECTL
- We made a ‘U-turn’ from ECTL# (and ECTL+) to bridge this gap

BTL Logics	$\mathsf{E}\Box\Diamond q$	$\mathsf{E}(\Box\Diamond q \wedge \Diamond\Box\neg q)$	$\mathsf{A}((p\mathcal{U}\Box q) \vee (s\mathcal{U}\Box\neg r))$	$\mathsf{A}\Diamond(\circ p \wedge \mathsf{E}\circ\neg p)$	One-pass Tableaux
$\mathcal{B}(\mathcal{U},\circ)$ (CTL)	X	X	X	X	This paper
$\mathcal{B}(\mathcal{U},\circ,\Box\Diamond)$ (ECTL)	$\checkmark$	X	X	X	This paper
$\mathcal{B}^{+}(\mathcal{U},\circ,\Box\Diamond)$ (ECTL^{+})	$\checkmark$	$\checkmark$	X	X	$\checkmark$
$\mathcal{B}^{+}(\mathcal{U},\circ,\mathcal{U}\Box)$ (ECTL$^{\#}$)	$\checkmark$	$\checkmark$	$\checkmark$	X	$\checkmark$
$\mathcal{B}^{*}(\mathcal{U},\circ)$ (CTL*)	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	X

One-pass Tableau Method for CTL - α- and β-Rules

- Standard but preserve context

$$(\wedge)\ \frac{\Sigma, \sigma_1 \wedge \sigma_2}{\Sigma, \sigma_1, \sigma_2} \qquad (\mathsf{Q}\Box)\ \frac{\Sigma, \mathsf{Q}\Box\sigma}{\Sigma, \sigma, \mathsf{Q}\circ\mathsf{Q}\Box\sigma}$$

$$(\vee)\ \frac{\Sigma, \sigma_1 \vee \sigma_2}{\Sigma, \sigma_1 \ \mid\ \Sigma, \sigma_2} \qquad (\mathsf{Q}\mathcal{U})\ \frac{\Sigma, \mathsf{Q}(\sigma_1 \mathcal{U} \sigma_2)}{\Sigma, \sigma_2 \ \mid\ \Sigma, \sigma_1, \mathsf{Q}\circ\mathsf{Q}(\sigma_1 \mathcal{U} \sigma_2)}$$

$$(\mathsf{Q}\mathcal{R})\ \frac{\Sigma, \mathsf{Q}(\sigma_1 \mathcal{R} \sigma_2)}{\Sigma, \sigma_1, \sigma_2 \ \mid\ \Sigma, \sigma_2, \mathsf{Q}\circ\mathsf{Q}(\sigma_1 \mathcal{R} \sigma_2)} \qquad (\mathsf{Q}\Diamond)\ \frac{\Sigma, \mathsf{Q}\Diamond\sigma}{\Sigma, \sigma \ \mid\ \Sigma, \mathsf{Q}\circ\mathsf{Q}\Diamond\sigma}$$

One-pass Tableau Method for CTL: Next-state Rule

- Responsible for obtaining AND-OR tree

$$(\mathrm{Q}\circ)\ \frac{\Sigma, \mathsf{A}\circ\sigma_1, \ldots, \mathsf{A}\circ\sigma_\ell, \mathsf{E}\circ\sigma'_1, \ldots, \mathsf{E}\circ\sigma'_k,}{\sigma_1, \ldots, \sigma_\ell, \sigma'_1 \ \& \ \ldots \ \& \ \sigma_1, \ldots, \sigma_\ell, \sigma'_k}$$

where Σ is a set of literals.

$$\mathsf{A}\circ\mathsf{A}(\mathbf{F}\,\mathcal{R}\,\neg q), \mathsf{E}\circ\mathsf{E}(p\,\mathcal{U}\,q) \wedge \mathsf{E}\circ\neg q$$

$(\wedge)$

$$\mathsf{A}\circ\mathsf{A}(\mathbf{F}\,\mathcal{R}\,\neg q), \mathsf{E}\circ\mathsf{E}(p\,\mathcal{U}\,q), \mathsf{E}\circ\neg q$$

$(\mathrm{Q}\circ)$

$$\mathsf{A}(\mathbf{F}\,\mathcal{R}\,\neg q), \mathsf{E}(p\,\mathcal{U}\,q) \qquad \mathsf{A}(\mathbf{F}\,\mathcal{R}\,\neg q), \neg q$$

One-pass Tableau Method for CTL: CONTEXT

- β^+-Rule – manages CONTEXT

$$(\mathsf{Q}\mathcal{U})^+ \quad \frac{\Sigma, \mathsf{Q}(\sigma_1 \mathcal{U} \sigma_2)}{\Sigma, \sigma_2 \;\mid\; \Sigma, \sigma_1, \mathsf{Q}{\circ}\mathsf{Q}((\sigma_1 \wedge \sim\Sigma')\mathcal{U}\sigma_2)}$$

where

$\Sigma' = \Sigma \setminus \{(\mathsf{A}\circ)^i \mathsf{A}\Box\sigma \mid i \geq 0 \text{ and } (\mathsf{A}\circ)^i \mathsf{A}\Box\sigma \in \Sigma\}$ and

$(\mathsf{A}\circ)^i$ stands for i times $\mathsf{A}\circ$.

$$\mathsf{A}(\mathbf{F}\,\mathcal{R}\,\neg q),\ \mathsf{E}(p\,\mathcal{U}\,q)$$

$(\mathsf{E}\mathcal{U})^+$

$$\mathsf{A}(\mathbf{F}\,\mathcal{R}\,\neg q),\ q \qquad\qquad \mathsf{A}(\mathbf{F}\,\mathcal{R}\,\neg q),\ p,\ \mathsf{E}{\circ}\,\mathsf{E}((p \wedge \mathsf{E}(\mathbf{T}\,\mathcal{U}\,q))\,\mathcal{U}\,q)$$

Tableaux Algorithm $\tau(\sigma)$

procedure SYSTEMATIC_TABLEAU(Σ_0) ▷ where Σ_0: set of CTL state formulae
  if Σ_0 is not uniform **then** T := Uniform_Tableau(Σ_0)
  while T has at least one expandable node **do**
    ▷ Invariant: Any expandable node of T is labelled by an uniform set
    Choose any node ℓ in T such that $\tau(\ell)$ is expandable ▷ $\tau(\ell)$ is uniform
    if there is no eventuality in $\tau(\ell)$ **then** $T := T[\ell \leftarrow$Uniform_Tableau($\tau(\ell)$)]
    else
      Eventuality_Selection($\tau(\ell)$)
      Apply_β^+-rule($\tau(\ell)$)
      Let ℓ_1, ℓ_2 the two children of ℓ
      for i = 1 .. 2 **do**
        if ℓ_i is expandable and $\tau(\ell_i)$ is not uniform **then**
          $T := T[\ell_i \leftarrow$Uniform_Tableau($\tau(\ell_i)$)]

Tableau Extension to ECTL

Two additional rules

$$(\mathsf{Q}\Box\Diamond)\ \frac{\Sigma, \mathsf{Q}\Box\Diamond\sigma}{\Sigma, \mathsf{Q}\Diamond\sigma, \mathsf{Q}\circ\mathsf{Q}\Box\Diamond\sigma} \qquad (\mathsf{Q}\Diamond\Box)\ \frac{\Sigma, \mathsf{Q}\Diamond\Box\sigma}{\Sigma, \mathsf{Q}\Box\sigma \ \mid\ \Sigma, \mathsf{Q}\Diamond\sigma, \mathsf{Q}\circ\mathsf{Q}\Diamond\Box\sigma}$$

that are based on these logical equivalences:

$$\mathsf{E}\Box\Diamond\sigma \equiv \mathsf{E}\Diamond\sigma \wedge \mathsf{E}\circ\mathsf{E}\Box\Diamond\sigma \qquad \mathsf{E}\Diamond\Box\sigma \equiv \mathsf{E}\Box\sigma \vee (\mathsf{E}\Diamond\sigma \wedge \mathsf{E}\circ\mathsf{E}\Diamond\Box\sigma)$$

$$\mathsf{A}\Box\Diamond\sigma \equiv \mathsf{A}\Diamond\sigma \wedge \mathsf{A}\circ\mathsf{A}\Box\Diamond\sigma \qquad \mathsf{A}\Diamond\Box\sigma \equiv \mathsf{A}\Box\sigma \vee (\mathsf{A}\Diamond\sigma \wedge \mathsf{A}\circ\mathsf{A}\Diamond\Box\sigma)$$

Conclusions and Future Work

- Context-based tableau extends naturally from linear-time to branching-time.
  - CTL tableau is much simpler than $\text{ECTL}^{\#}$ one, where two types of context (outer and inner) are used.
- Context-based tableau keeps the duality with sequent calculus.
  - Application in certified model checking: sequent proofs as certificates.
- Intuitive tableau method for CTL and ECTL.
  - Well suited for the automation and amenable for the implementation.