Dynamic Branching in Qualitative Constraint Networks via Counting Local Models

Michael Sioutis Diedrich Wolter

Information Systems and Applied Computer Sciences
Bamberg University

September 25, 2020

Qualitative Spatial & Temporal Reasoning

- QSTR is a major field of study in Knowledge Representation & Reasoning.
- QSTR abstracts from numerical quantities of space and time by using qualitative descriptions instead (e.g., *precedes*, *contains*, *is left of*).

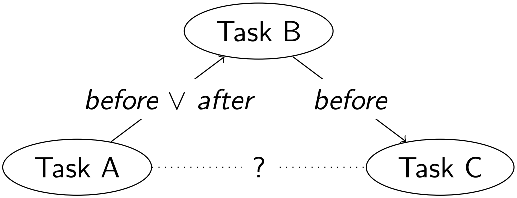


Figure: A qualitative temporal configuration

Allen's Interval Algebra (IA) Constraint Language

precedes	p	pi
meets	m	mi
overlaps	o	oi
starts	s	si
during	d	di
finishes	f	fi
equals	eq	

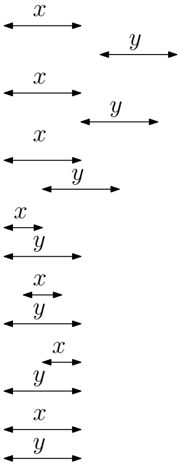
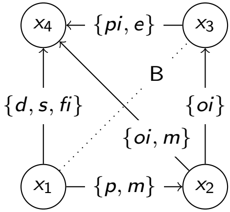


Figure: The thirteen base relations of Interval Algebra

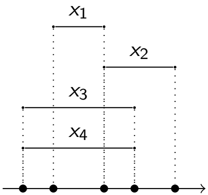
Qualitative Constraint Network (QCN)

Definition

A QCN *is a pair* $\mathcal{N} = (V, C)$ *where* V *is a non-empty finite set of variables, and* C *a mapping* $C : V \times V \to 2^{\mathsf{B}}$.



(a) A satisfiable QCN $\mathcal{N}$



(b) A solution σ of $\mathcal{N}$

Figure: Figurative examples of QCN terminology using Interval Algebra

Fundamental Reasoning Problems of QCNs

Definition

The satisfiability checking problem of a QCN $\mathcal{N}$ *is deciding whether* $\mathcal{N}$ *admits a solution.*

- Deciding the satisfiability of a QCN is NP-complete in general.

Definition

*The minimal labeling problem (*MLP*) of a* QCN $\mathcal{N}$ *is finding the strongest implied constraints of* $\mathcal{N}$*.*

- The MLP is polynomial-time Turing reducible to the satisfiability checking problem [GS93].

Solving via backtracking search

Let us consider a QCN $\mathcal{N}$, we tackle it as follows.

- Every relation r forming a constraint in $\mathcal{N}$ is split into subrelations $r' \subseteq r$.
- These subrelations r' belong to a set of relations $\mathcal{A}$ over which the QCN becomes tractable [RN07].
- After every refinement of a relation r into one of its subrelations r', a validity check is performed:
  - if the refinement is valid, we proceed with the next one;
  - if the refinement is not valid, we backtrack to the previous one.

Backtracking search algorithm: branching

Algorithm 1: BacktrackingSearch($\mathcal{N}$, G, $\mathcal{A}$, f = null)

in/out : A QCN $\mathcal{N} = (V, C)$.
in : A graph $G = (V, E)$, a subset $\mathcal{A} \subseteq 2^{\mathsf{B}}$, and a function f $\in$ {max, min, avg, sum} or null.
output : A refinement of $\mathcal{N}$ with respect to G over $\mathcal{A}$, or $\perp^V$.

```
begin
    N ← ◇_G(N);
    if ∅ ∈ ◇_G(N) then                                    // Validity check
        return ⊥^V;
    if ∀{v, v'} ∈ E, C(v, v') ∈ A then
        return N;
    (v, v') ← {v, v'} ∈ E such that C(v, v') ∉ A;
    /* Split r into subrelations r' ∈ A, and dynamically prioritize the selection of those
       subrelations according to function f, where null denotes a static selection    */
    foreach r ∈ Selection(N, G, A, (v, v'), f) do
        result ← Refinement(N_[v,v']/r, G, A, f);
        if result ≠ ⊥^V then
            return result;
    return ⊥^V;
```

Problems with static selection

The static strategy

- assumes a uniform use of relations in QCNs
- does not exploit any structure that may exist in QCNs

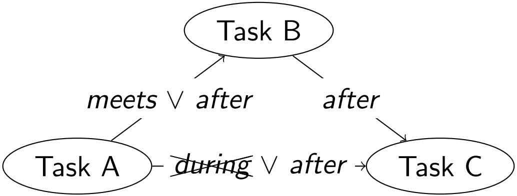


Figure: The static weighting scheme in the literature dictates that relation *during* is less restrictive than relation *after* in general for the IA calculus and, hence, *during* should be preferred over *after* in branching decisions [BM96, Figure 9], but in the above simplified QCN *during* is not feasible

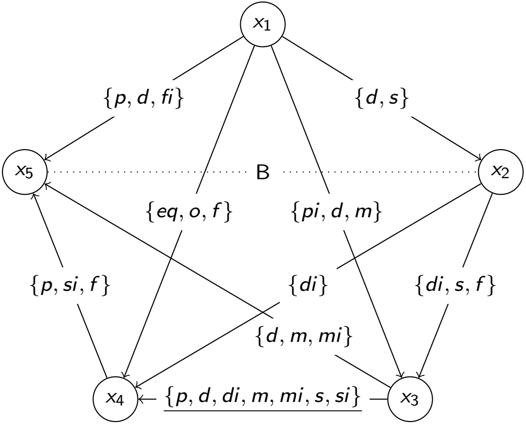
Leveraging local models for dynamic branching

Definition (local models)

Given a QCN $\mathcal{N} = (V, C)$, a graph $G = (V, E)$, and a constraint $C(v, v')$ with $\{v, v'\} \in E$, the *local models* of a base relation $b \in C(v, v')$ are the atomic refinements $\mathcal{S} = (V', C')$ of $\mathcal{N}{\downarrow}_{V'}$, with $V' = \{v, v', u\}$, such that $\{v, u\}, \{u, v'\} \in E$ and $C(v, v') = \{b\}$.

- We count how many times b participates in the atomic refinements of each triangle in G involving v and v'
- In that sense, our approach can be seen as being similar to a counting-based one for CSPs [PQZ12]

Example



b	$\#l$-models
p	6
d	6
di	7
m	4
mi	5
s	3
si	3

Figure: Given the above QCN $\mathcal{N} = (V, C)$ of IA, a partition of $C(x_3, x_4)$ with respect to the subset $\mathcal{H}_{\text{IA}}$ [NB95] is $\{\{mi\}, \{di, si\}, \{p, m\}, \{d, s\}\}$; the number of local models for each base relation is shown in the table

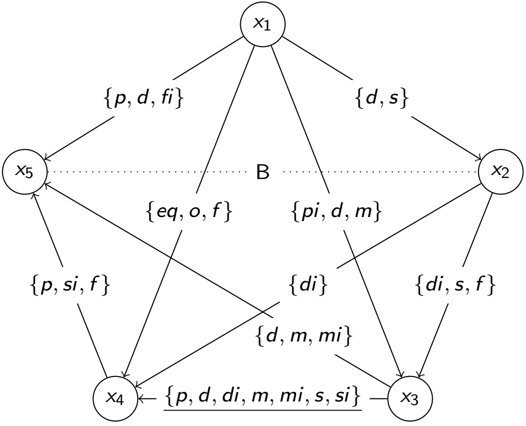
Dynamic selection strategies

We devise the following strategies for choosing a subrelation from a given set of subrelations:

- **dynamic_f**: for each subrelation r' find the f count of local models among each base relation $b \in r'$, where f $\in \{\mathrm{max}, \mathrm{min}, \mathrm{avg}, \mathrm{sum}\}$, then choose the subrelation for which the highest such count was obtained.

In short dynamic_max, dynamic_min, dynamic_avg, and dynamic_sum prioritize the subrelation with the best *most*, *least*, *on avegare*, and *in aggregate* supportive base relation respectively.

Example (revisited)



b	#l-models
p	6
d	6
di	7
m	4
mi	5
s	3
si	3

Figure: Strategies *dynamic*_avg and *dynamic*_sum would prioritize subrelation $\{p, m\}$, and strategies *static* [BM96], *dynamic*_max, and *dynamic*_min would prioritize subrelations $\{d, s\}$, $\{di, si\}$, and $\{mi\}$ respectively

Evaluation (1/2)

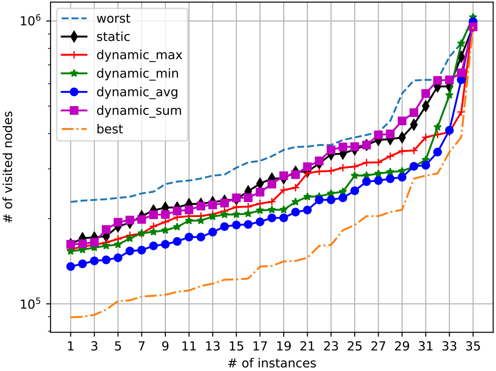


Figure: Insight into the 0.5th percentile of most difficult instances of model $H(n = 40, d)$ [Neb97] for each strategy

Evaluation (2/2)

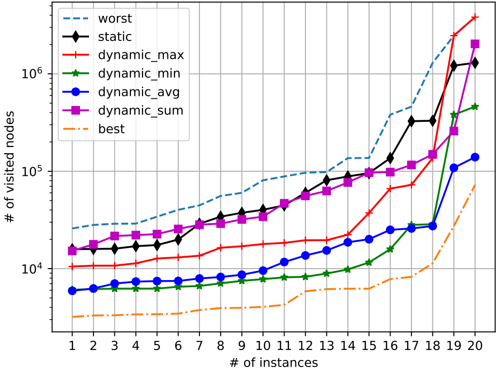


Figure: Insight into the 0.5th percentile of most difficult instances of model $\mathrm{BA}(n = 80, m, 3\mathrm{CNF})$ [SCK16] for each strategy

Conclusion and Future Work

- We introduced and evaluated dynamic branching strategies for solving QCNs via backtracking search.
- Our approach is adaptive; it preserves most of the (global) solutions by determining what proportion of local solutions agree with a branching decision.
- We can obtain 5 times better performance for structured instances of IA, and up to 20% gains for random ones.
- We aim to devise selection protocols that choose among different strategies, and implement them as portfolios.
- More sophisticated dynamic heuristics could be developed by engaging larger parts of an instance.

Thank you for your interest and attention!

References

Peter van Beek and Dennis W. Manchak. "The design and experimental analysis of algorithms for temporal reasoning". In: *J. Artif. Intell. Res.* 4 (1996), pp. 1–18 (cit. on pp. 8, 12).

Martin Charles Golumbic and Ron Shamir. "Complexity and Algorithms for Reasoning about Time: A Graph-Theoretic Approach". In: *J. ACM* 40 (1993), pp. 1108–1133 (cit. on p. 5).

Bernhard Nebel and Hans-Jürgen Bürckert. "Reasoning about Temporal Relations: A Maximal Tractable Subclass of Allen's Interval Algebra". In: *J. ACM* 42 (1995), pp. 43–66 (cit. on p. 10).

Bernhard Nebel. "Solving Hard Qualitative Temporal Reasoning Problems: Evaluating the Efficiency of Using the ORD-Horn Class". In: *Constraints* 1 (1997), pp. 175–190 (cit. on p. 13).

Gilles Pesant, Claude-Guy Quimper, and Alessandro Zanarini. "Counting-Based Search: Branching Heuristics for Constraint Satisfaction Problems". In: *J. Artif. Intell. Res.* 43 (2012), pp. 173–210 (cit. on p. 9).

Jochen Renz and Bernhard Nebel. "Qualitative Spatial Reasoning Using Constraint Calculi". In: *Handbook of Spatial Logics*. Springer-Verlag, 2007, pp. 161–215 (cit. on p. 6).

Michael Sioutis, Jean-François Condotta, and Manolis Koubarakis. "An Efficient Approach for Tackling Large Real World Qualitative Spatial Networks". In: *Int. J. Artif. Intell. Tools* 25 (2016), pp. 1–33 (cit. on p. 14).