Temporal Modalities in Answer Set Programming

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- Stephane Demri (CNRS, ENS Paris-Saclay, FR)
- Laura Bozzelli (U. Naples, IT)

Motivation

- Context: temporal reasoning with transition systems.
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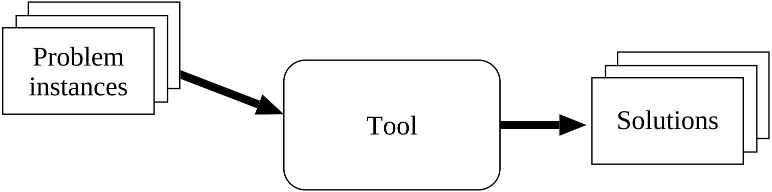

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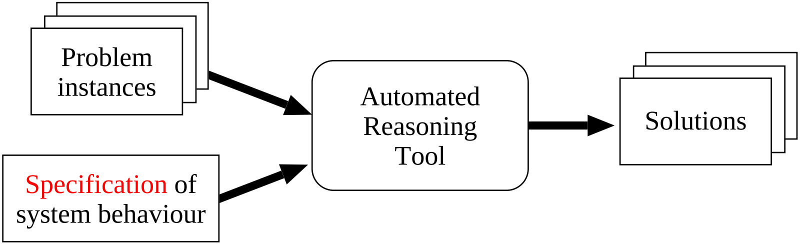

Keypoint: representation

Which are the desirable properties of a good KR?

1. Simplicity
2. Natural understanding: clear semantics
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Example: automata satisfy everything, but lack elaboration

Temporal ASP

- Elaboration tolerance for action domains:

  Representing Action and Change by Logic Programs [Gelfond & Lifschitz 93] use Answer Set Programming (ASP)

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- However, ASP has no temporal constructs

Transition systems in ASP

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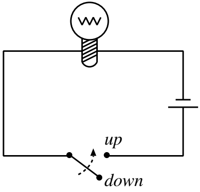


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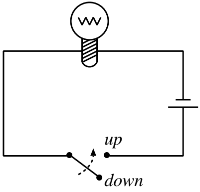


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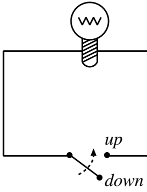


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- Is there a reachable state with *up* and *down* false?
- Once *up* becomes true, does it remain so forever?
- The switch cannot be closed infinitely often without eventually damaging the lamp

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These topics typically covered by (Modal) Temporal Logics

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A simple and well-known case

Linear-time Temporal Logic (LTL)

$\square$ (forever), $\Diamond$ (eventually), $\circ$ (next), $\mathbf{U}$ (until)

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- ✓ Model checking and verification of reactive systems
- ✓ Many uses in AI: planning, ontologies, multi-agent systems, . . .

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- ✗ NMR attempts for LTL: limited syntax, only for queries, control rules, etc. Not really embodied in LTL

Our proposal

Temporal Equilibrium Logic (TEL) [C_&Pérez 07]

TEL = ASP + LTL

- ASP: logical characterisation Equilibrium Logic [Pearce 96]
- LTL: We add temporal operators $\square$, $\Diamond$, $\circ$, **U**, **R**.

Result: Temporal Stable Models for any arbitrary LTL theory.

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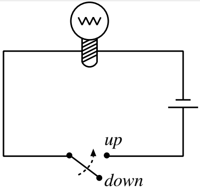
- but we can arbitrarily close it at any moment.

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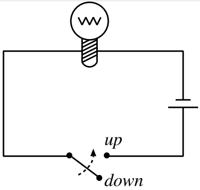

$up \vee down$	Initially
$\Box(\circ up \leftarrow up \wedge \neg\circ down)$	Inertia
$\Box(\circ down \leftarrow down \wedge \neg\circ up)$	Inertia
$\Box(up \vee \neg up)$	Choice

Idea: LTL syntax, but keeping ASP semantics

Equilibrium Logic

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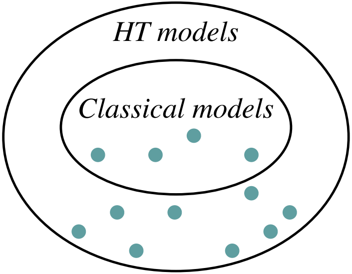

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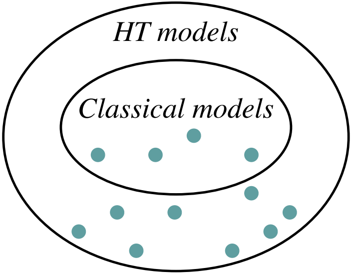

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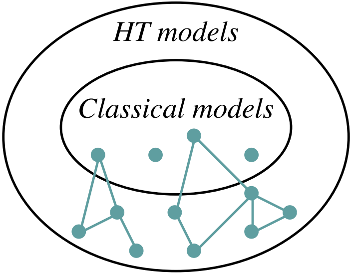

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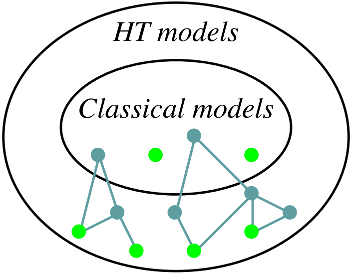

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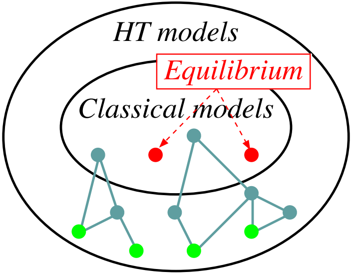

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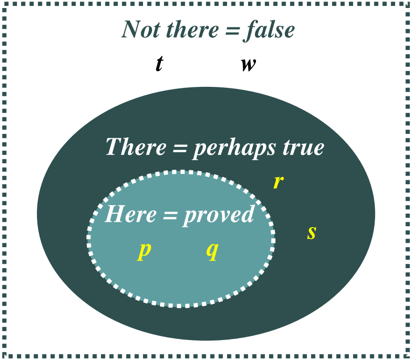
- Example: $H = \{p, q\}, T = \{p, q, r, s\}$. Intuition:



- When $H = T$ we have a classical model.

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Satisfaction of formulas

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- $\langle H, T \rangle \models \varphi \rightarrow \psi$ if both
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  - $\langle H, T \rangle \models \varphi$ implies $\langle H, T \rangle \models \psi$

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- Negation $\neg F$ is defined as $F \rightarrow \bot$
- $\langle H, T \rangle \models \varphi$ implies $T \models \varphi$ (proved implies potentially true)

Equilibrium models

Definition (Equilibrium/stable model)

A model $\langle T, T \rangle$ of Γ is an equilibrium model iff

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In other words, all assumptions T are eventually proved H

(Linear) Temporal Equilibrium Logic

- Syntax = propositional plus
  - $\Box\varphi$ = "forever" φ
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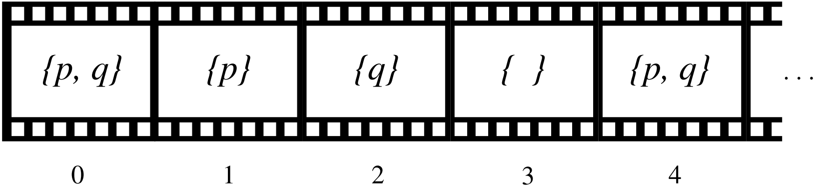
- As we had with Equilibrium Logic:
  1. A monotonic underlying logic: Temporal Here-and-There (THT)
  2. An ordering among models. Select minimal models.

Sequences

- In standard LTL, interpretations are ∞ sequences of sets of atoms



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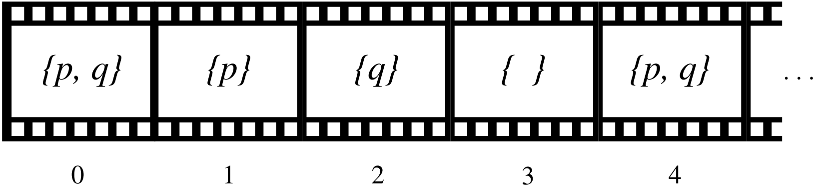
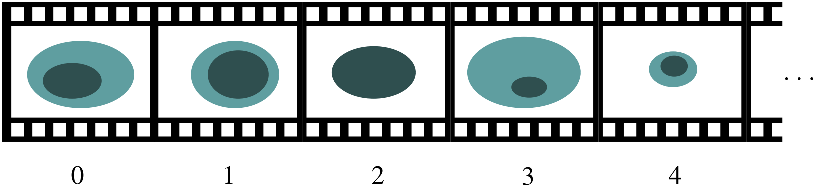


- In THT we will have ∞ sequences of HT interpretations



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- We define an ordering among sequences $\mathbf{H} \leq \mathbf{T}$ when

$$\begin{array}{ccccccccccc} T_0 & \longrightarrow & T_1 & \longrightarrow & T_2 & \longrightarrow & \dots & \longrightarrow & T_i & \longrightarrow & \dots \\ \cup\vert & & \cup\vert & & \cup\vert & & & & \cup\vert & & \\ H_0 & \longrightarrow & H_1 & \longrightarrow & H_2 & \longrightarrow & \dots & \longrightarrow & H_i & \longrightarrow & \dots \end{array}$$

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- An interpretation $M = \langle \mathbf{H}, \mathbf{T} \rangle$ satisfies α at situation i, written $M, i \models \alpha$

α	$M, i \models \alpha$ when ...
an atom p	$p \in H_i$
$\wedge, \vee$	as usual
$\varphi \rightarrow \psi$	$\mathbf{T}, i \models \varphi \rightarrow \psi$ in LTL and $\langle \mathbf{H}, \mathbf{T} \rangle, i \models \varphi$ implies $\langle \mathbf{H}, \mathbf{T} \rangle, i \models \psi$

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α	$M, i \models \alpha$ when ...
$\circ\varphi$	$(M, i{+}1) \models \varphi$
$\Box\varphi$	$\forall j \geq i, \quad M, j \models \varphi$
$\Diamond\varphi$	$\exists j \geq i, \quad M, j \models \varphi$
$\varphi \ \mathbf{U} \ \psi$	$\exists j \geq i, \quad M, j \models \psi$ and $\forall k$ s.t. $i \leq k < j, \quad M, k \models \varphi$
$\varphi \ \mathbf{R} \ \psi$	$\forall j \geq i, \quad M, j \models \psi$ or $\exists k, i \leq k < j, \quad M, k \models \varphi$

- M is a model of a theory Γ when $M, 0 \models \alpha$ for all $\alpha \in \Gamma$

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- $\circ\varphi$

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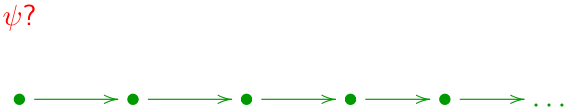

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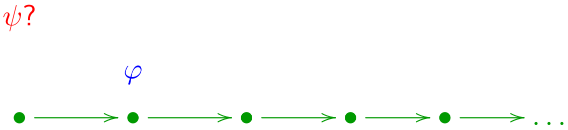

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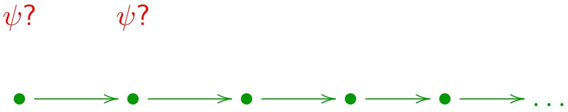

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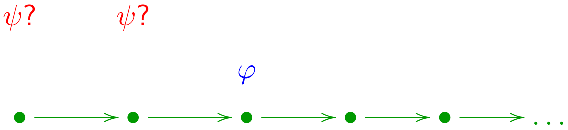

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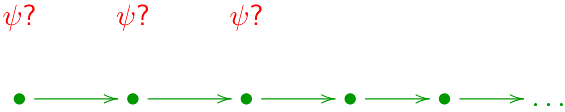

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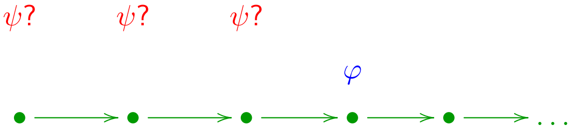

Temporal Here-and-There (THT)

- Some valid THT formulas:

$$
\begin{array}{rcl}
\Diamond\varphi & \leftrightarrow & \top \;\mathbf{U}\; \varphi \\
\Box\varphi & \leftrightarrow & \bot \;\mathbf{R}\; \varphi \\
\circ(\varphi \otimes \psi) & \leftrightarrow & \circ\varphi \otimes \circ\psi \\
\varphi \;\mathbf{U}\; \psi & \leftrightarrow & \psi \vee (\varphi \wedge \circ(\varphi \;\mathbf{U}\; \psi)) \\
\varphi \;\mathbf{R}\; \psi & \leftrightarrow & \psi \wedge (\varphi \vee \circ(\varphi \;\mathbf{R}\; \psi)) \\
\varphi \;\mathbf{W}\; \psi & \leftrightarrow & \varphi \wedge (\psi \rightarrow \circ(\varphi \;\mathbf{W}\; \psi)) \\
\neg(\varphi \;\mathbf{U}\; \psi) & \leftrightarrow & \neg\varphi \;\mathbf{R}\; \neg\psi \\
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For $\otimes = \wedge, \vee, \rightarrow, \mathbf{U}, \mathbf{R}$.

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\end{array}$$

For $\otimes = \wedge, \vee, \rightarrow, \mathbf{U}, \mathbf{R}$.

- Axiomatization of THT [Balbiani & Diéguez 16]

Temporal Equilibrium Models

Definition (Temporal Equilibrium Model)

of a theory Γ is a model $M = \langle \mathbf{T}, \mathbf{T} \rangle$ of Γ such that there is no $\mathbf{H} < \mathbf{T}$ satisfying $\langle \mathbf{H}, \mathbf{T} \rangle, 0 \models \Gamma$. □

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- Temporal Equilibrium Logic (TEL) is the logic induced by temporal equilibrium models.

Definition (Temporal Stable Model)

$\mathbf{T}$ is a temporal stable model of a theory Γ iff $\langle \mathbf{T}, \mathbf{T} \rangle$ is a temporal equilibrium model of Γ. □

Some examples

- Example 1: TEL models of $\Box(\neg p \rightarrow \circ p)$. It's like an infinite program:

$$\begin{array}{rcl} \neg p & \rightarrow & \circ p \\ \neg \circ p & \rightarrow & \circ^2 p \\ \neg \circ^2 p & \rightarrow & \circ^3 p \\ & \vdots & \end{array}$$

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- TEL models have the form

$$\begin{array}{cccccc} \emptyset & p & \emptyset & p & \emptyset & \\ \bullet \longrightarrow & \bullet \longrightarrow & \bullet \longrightarrow & \bullet \longrightarrow & \bullet \longrightarrow & \ldots \end{array}$$

corresponding to LTL models of $\neg p \wedge \square(\neg p \leftrightarrow \circ p)$.

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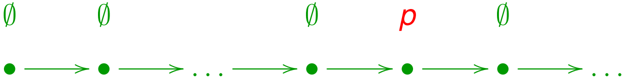

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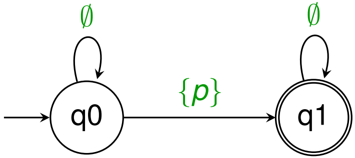

- Answer: using Büchi automata. An infinite-length word is accepted iff it visits some acceptance state infinitely often

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- Therefore, $\Box\Diamond p$ alone has no TEL models.

Some examples

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- In fact, if we add (EM) for all atoms, TEL collapses into LTL

Some examples

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$$\begin{aligned} & \Box\big((\neg\circ p \rightarrow p) \wedge (\circ p \rightarrow p)\big) \\ \equiv\ & \Box\big(\underbrace{\neg\circ p \vee \circ p}_{\top} \rightarrow p\big) \\ \equiv\ & \Box p \end{aligned}$$

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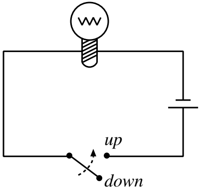
- So LTL models make p true forever, but we won't get TEL models!
- We can build a strictly smaller model with **H** where from some point on **T**, p becomes false forever

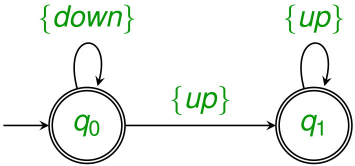
T	p	p		p	p	p
	$\parallel$	$\parallel$		$\parallel$	$\cup$	$\cup$
H	p	p		p	$\emptyset$	$\emptyset$
	$\bullet \longrightarrow$	$\bullet \longrightarrow$	$\ldots \longrightarrow$	$\bullet \longrightarrow$	$\bullet \longrightarrow$	$\bullet \longrightarrow \ldots$

Some examples

- Example 5: lamp switch again



	$up \vee down$	Initially
$\Box(up \wedge \neg \circ down \rightarrow$	$\circ up)$	Inertia
$\Box(down \wedge \neg \circ up \rightarrow$	$\circ down)$	Inertia
	$\Box(up \vee \neg up)$	Choice



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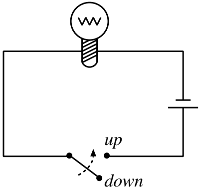

	$up \lor down$	Initially
$\Box(up \land \neg\circ down$	$\rightarrow \circ up)$	Inertia
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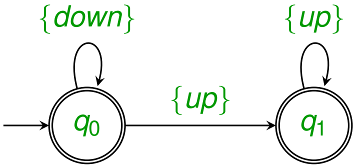
	$\Box(up \lor \neg up)$	Choice



We never get $up \land down$
Once up is true, it remains so forever

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- Temporal equilibrium models of φ are in one-to-one relation to Equilibrium Models of the first order formula $Kamp[\varphi]$
- FO-Equilibrium Logic is the most general logical characterisation of ASP

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- THT can be encoded into LTL, adding auxiliary atoms using the same translation of $\rightarrow$ from HT to classical logic
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Example

THT		**LTL**
$\Box(down \wedge \neg\circ up \rightarrow \circ down)$	$\equiv$	$\Box(up' \rightarrow up) \wedge \Box(down' \rightarrow down)$
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- THT-satisfiability = PSPACE-complete [C_ & Demri 11]
  TEL-satisfiability = EXPSPACE-complete [Bozzelli & Pearce 15]

Automata-based methods

[C_ & Demri 2011]

Definition (Automata Based Computation Method)

LTL (i. e. total) models which do not have a strictly smaller $\langle H, T \rangle$

$$\mathcal{A}_\varphi \quad \otimes \quad \overline{h(\mathcal{A}_{\varphi'})}$$

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- Büchi automata are closed w.r.t. complementation and intersection

Example

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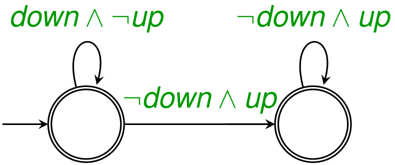

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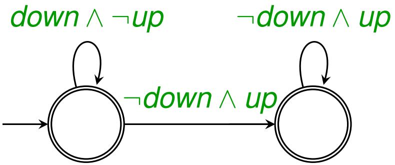

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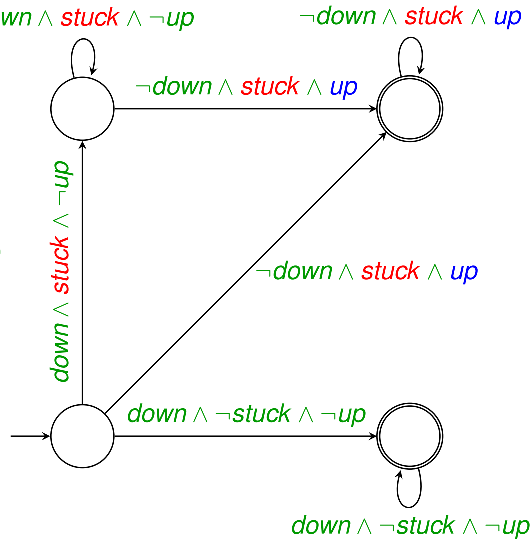

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- Adding past operators to LTL: same expressiveness but exponentially more succint [Markey 03]

Finite traces and past operators

Past	$\bullet$	for *previous*	*Future*	$\circ$	for *next*
	S	for *since*		**U**	for *until*
	T	for *trigger*		**R**	for *release*
				W	for *while*

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					$\mathbf{W}$	for *while*

$\blacksquare\varphi$	$\stackrel{def}{=}$	$\bot\,\mathbf{T}\,\varphi$	*always before*	$\Box\varphi$	$\stackrel{def}{=}$	$\bot\,\mathbf{R}\,\varphi$	*always after*
$\blacklozenge\varphi$	$\stackrel{def}{=}$	$\top\,\mathbf{S}\,\varphi$	*eventually before*	$\Diamond\varphi$	$\stackrel{def}{=}$	$\top\,\mathbf{U}\,\varphi$	*eventually after*
$\mathbf{I}$	$\stackrel{def}{=}$	$\neg\bullet\top$	*initial*	$\mathbf{F}$	$\stackrel{def}{=}$	$\neg\top$	*final*
$\widehat{\bullet}\varphi$	$\stackrel{def}{=}$	$\bullet\varphi\vee\mathbf{I}$	*weak previous*	$\widehat{\circ}\varphi$	$\stackrel{def}{=}$	$\varphi\vee\mathbf{F}$	*weak next*

Finite traces and past operators

- Satisfaction of formulas introduces conditions on trace limits on the past ($i \geq 0$) and the future ($i < \lambda$)

α	$M, i \models \alpha$ when ...
$\circ\varphi$	$i + 1 < \lambda$ and $(M, i{+}1) \models \varphi$
$\widehat{\circ}\varphi$	$i + 1 = \lambda$ or $(M, i{+}1) \models \varphi$
φ **U** ψ	$\exists j : i \leq j < \lambda,\quad M, j \models \psi$ and $\forall k$ s.t. $i \leq k < j,\quad M, k \models \varphi$
$\bullet\varphi$	$i > 0$ and $(M, i{-}1) \models \varphi$
$\widehat{\bullet}\varphi$	$i = 0$ or $(M, i{-}1) \models \varphi$
φ **S** ψ	$\exists j : 0 \leq j \leq i,\quad M, j \models \psi$ and $\forall k$ s.t. $j < k \leq i,\quad M, k \models \varphi$

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$\bullet\varphi$	$i > 0$ and $(M, i{-}1) \models \varphi$
$\widehat{\bullet}\varphi$	$i = 0$ or $(M, i{-}1) \models \varphi$
$\varphi \ \mathbf{S} \ \psi$	$\exists j : 0 \leq j \leq i, \quad M, j \models \psi$ and $\forall k$ s.t. $j < k \leq i, \quad M, k \models \varphi$

- When $\lambda = \omega$ we get (infinite-traces) TEL as before

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A temporal rule is either:

- an initial rule $B \rightarrow A$
- a dynamic rule $\widehat{\circ}\,\Box(B \rightarrow A)$
- a fulfillment rule $\Box(\,\Box\, p \rightarrow q\,)$ or $\Box(\,p \rightarrow \Diamond\, q\,)$

where $B = b_1 \wedge \cdots \wedge b_n$ with $n \geq 0$, $A = a_1 \vee \cdots \vee a_m$ with $m \geq 0$
b_i, a_j = temporal literals for dynamic rules
b_i, a_j = regular literals $a, \neg a$ for initial rules
p, q = atoms

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Definition (Temporal rule)

A temporal rule is either:

- an initial rule $B \to A$
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A temporal logic program is a set of temporal rules.

Syntactic Fragment

- An interesting fragment are present-centered programs
  - initial rule $B \rightarrow A$
  - dynamic rule $\widehat{\circ}\,\square(B \rightarrow A)$
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  present-centered = A does not contain temporal operators

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- Example of present-centered program:

	$up \vee down$	Initially
$\widehat{\circ}\Box(\bullet up \wedge \neg down \;\rightarrow$	$up)$	Inertia
$\widehat{\circ}\Box(\bullet down \wedge \neg up \;\rightarrow$	$down)$	Inertia
	$\Box(up \vee \neg up)$	Choice

Syntactic Fragment

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  Temporal extension of ASP solver `clingo`

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```
#program initial.
up;down.

#program dynamic.
up   :- 'up,   not down.
down :- 'down, not up.

#program always.
{up}.
```

Syntactic Fragment

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  - α may only contain past operators
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  and none of them contains $\rightarrow$

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can be expressed in `telingo` as:

```
:- shoot, &tel { <* unloaded & < <? shoot }.
```

Beyond LTL

- In [Bosser et al. 18, C_ et al. 19] we extend TEL and TEL_f to the syntax of Linear Dynamic Logic (LDL) [De Giacomo & Vardi 13]
- DEL = LDL + ASP.

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  elevator moving in a unique direction until the call is served
- [C_ et al, ECAI 20] LDL operators implemented in `telingo` (only in constraints)

```
#program initial.
:- not &del{ *( (*up + *down) ;; ?ready ;; serve)
               ;; *wait .>? &final }.
```

Beyond LTL

- ☞ In [C_ et al, ICLP 20] (🕘Tomorrow 18:15) we introduce metric operators

$$\Box(red \wedge green \rightarrow \bot)$$
$$\Box(\neg green \rightarrow red)$$
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- We extended this for intervals (only discrete by now).

Conclusions

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- TEL = suitable framework for temporal reasoning + ASP
- Simple semantics thanks to just merging two logical formalisms: Equilibrium Logic + LTL.
- Implementations: `telingo`, `abstem`, `stelp`
- It constitutes a new open field. Many open topics …

Open topics

- Open theoretical problems:
  - Kamp's theorem: monadic EL(<) can be transformed into THT? (possibly not)
  - Interdefinability of operators
  - Can temporal stable models be captured by LTL?
- Finite traces: axiomatisation, automata-based methods, grounding
- New syntactic subclasses with satisfiability lower than EXPSPACE [Bozzelli & Pearce 15]
- Planning tool. Compare to planners using LTL control knowledge like TLPlan [Bacchus & Kabanza 00].
- Encoding action languages

Temporal Modalities in Answer Set Programming

Pedro Cabalar

Thank you for your attention!

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Bozen-Bolzano, Italy